B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5DSE21 (Probability & Statistics)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Prove that the distribution function *F(x)* of a random variable *X* has left discontinuity at a point $x=a$ when $P(X=a)>0$. [5]

(b) Write down the probability mass function of negative binomial distribution. Also find the mean of this distribution. [2+3]

(c) A point is chosen at random on a semi-circle having centre at the origin and radius unity and projected on the diameter. Prove that the distance of the point of projection from the centre has probability density $\frac{1}{\pi\sqrt{1-x^2}}, -1<x<1$ and zero, elsewhere. [5]

(d) If X be a normal (m,σ)-distribution, find the probability density function of $\frac{1}{2}\left(\frac{X-m}{\sigma}\right)^2$. [5]

(e) For what value of k, $f(x,y)$ represents the probability density function of two dimensional continuous random variable (X,Y) ,where [2+3]

$$f(x,y)=k(4-2x+y);\ 0<x<3,\ 2<y<4$$
$$= 0;\ \text{elsewhere}.$$

Also find $P(X<2/Y<3)$.

(f) State Bernoulli's theorem of convergence in probability. Show that Bernoulli's theorem can be obtained as a particular case of Law of large numbers for equal components. [5]

(g) Given that $X_1, X_2, ..., X_n$ are independently distributed normal (0, σ) distribution. Show that $\sqrt{\frac{\pi}{2}}\sum_{i=1}^{n}\frac{1}{n}|X_i|$ is an unbiased estimator of σ. [5]

(h) The heights of 10 males of a normal population are found to be 70, 67, 62, 67, 61, 68, 70, 64, 65, 66 inches. Is it reasonable to believe that average height is greater than 64 inches? Test at 5% significance level assuming that for 9 degrees of freedom $P(t>1.83)=0.05$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) If the random variable X follows Poisson distribution with parameter λ, show that [5] the distribution function of X is given by $F(x)=\frac{1}{\Gamma(x+1)}\int_{\lambda}^{\infty} e^{-t}t^{x}dt$, $x=0,1,2,...$

(ii) Define mathematical expectation of a random variable X. If the probability density function of a random variable X be $f(x)=\frac{1}{\pi(1+x^2)}, -\infty<x<\infty$, show that $E(X)$ does not exist. [2+3]

(b) (i) The radius X of a circle has uniform distribution in the interval (1,3). Find the mean and variance of the area of the circle. [2+3]

(ii) The probability density function of a random variable X is given by $f(x)=ae^{-ax}; a>0$ and $0<x<\infty$. Obtain the moment generating function of X and hence find $E(X^n)$. [3+2]

(c) (i) The joint probability density function of two dimensional random variables X and Y is [3+2]

$$f(x,y)=k(1-x-y); x\geq 0, y\geq 0, x+y\leq 1$$
$$=0\ ,\ \text{elsewhere}$$

where k is a constant. Find (i) the mean value of Y when X=1/2 (ii) the covariance of X and Y.

(ii) Define moment generating function of bivariate continuous random variable (X,Y). If the bivariate random variable (X,Y) follows normal distribution with parameters (0,0,1,1,ρ), then show that its moment generating function is given by $M_{XY}(t_1,t_2)=e^{\frac{1}{2}(t_1^2+t_2^2+2\rho t_1 t_2)}$. [1+4]

(d) (i) If $\overline{X}$ be the sample mean of the sample ($X_1, X_2......X_n$) drawn from the population with mean μ and variance σ^2, show that $E(\overline{X})=\mu, \text{var}(\overline{X})=\sigma^2/n$ and $E\left(\frac{n}{n-1}M_2\right)=\sigma^2$ where M_2 is the sample central moment of order 2. [2+2+2]

(ii) The probability density function of a random variable X is given by [2+2]

$$f(x;\theta)=\frac{1}{\theta}, 0\leq x<\theta$$
$$=0, \text{elsewhere}$$

You have to test the null hypothesis $H_0:\theta=1$ against the alternative hypothesis $H_1:\theta=2$ by means of a single observed value of x. What would be the probability of type-I and type-II errors, if you choose the interval $x\geq 0.5$ as the critical region?

(e) (i) What do you mean by interval estimation? Obtain the $100(1-\alpha)\%$ confidence interval for the population variance of a normal distribution assuming the mean of the distribution. [1+4]

(ii) A sample of size n is drawn from a normal (m,σ) population, the standard [5] deviation σ being known. Discuss how you could test the hypothesis $H_0 : m = m_0$ against an alternative hypothesis $H_1 : m = m_1 > m_0$ at a given level of significance α.

B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5DSE22 (Portfolio Optimization)

Time: 3 Hours   Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) What do you mean by investment? [5]

(b) Write a brief discussion about measures of return and risk. [5]

(c) What specific securities should be purchased for the portfolio? [5]

(d) Write a short note about investment constraint. [5]

(e) Discuss leverage and its effect on the CML. [5]

(f) Explain why the line from the RFR that is tangent to the efficient frontier defines the dominant set of portfolio possibilities. [5]

(g) In the empirical testing of the CAPM, what are two major concerns? Why are they important? [5]

(h) Draw an ideal SML. Based on the early empirical results, what did the actual risk-return relationship look like relative to the ideal relationship implied by the CAPM? [5]

2. Answer any three questions: 10×3 = 30

(a) The rate of return expectations for the common stock of a company during the next year are [6+2+2]

Possible rate of return	Probability
-0.10	0.25
0.00	0.15
0.10	0.35
0.25	0.25

(i) Compute the expected return $\left[E(R_i)\right]$ on this investment, the variance of this return $\left[\sigma^2\right]$ and its standard deviation $\left[\sigma\right]$.

(ii) Under what conditions can the standard deviation be used to measure the relative risk of two investments?

(iii) Under what conditions must the coefficient of variation be used to measure the relative risk of two investments?

(b) Derive mean-variance portfolio optimization of Markowitz model. [10]

(c) Write a brief discussion about zero beta model. What is a zero-beta asset and how does its use impact the CML? [7+3]

(d) (i) Calculate the expected (required) return for each of the following stocks when the risk-free rate is0.08 and you expect the market return to be 0.14. [5]

Stock	beta
A	1.72
B	1.14
C	0.76
D	0.44
E	0.03
F	-0.79

(ii) Assuming that the empirical proxy for the market portfolio is not a good proxy, what factors related to the CAPM will be affected? Briefly discuss why it is important for beta coefficients to be stationary over time. [2+3]

(e) Suppose your first job pays you $28,000 annually. What percentage should your cash reserve contain? How much life insurance should you carry if you are unmarried? If you are married then how much life insurance should you carry with two young children? [3+3+4]

B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5DSE23 (Boolean Algebra and Automata Theory)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

a) (i) Write a short note on Halting problem. [2]

(ii) Prove that in a bounded distributive lattice an element can have at most one complement. [3]

(b) (i) Define atom and give an example of it. [2]

(ii) In a finite Boolean algebra, every non zero element can be uniquely expressed as the sum of all atoms. [3]

(c) If f is a function of three Boolean variables x, y, z defined by $f(x,y,z) = xy + y'$, express f in disjunctive normal form. [5]

(d) (i) Define modular lattices and give an example of it. [2]

(ii) Prove that a lattice is modular iff it satisfies $x \cup (y \cap (x \cup z)) = x \cup (z \cap (x \cup y))$. [3]

(e) A committee of three persons A, B, C decides proposals by a majority of votes. A has a voting weight 3, B has a voting weight 2, and C has a voting weight 1. Design a simple circuit so that the light will glow when a majority of votes is cast in favour of the proposals. [5]

(f) Prove that context free languages are closed under union, concatenation, and reversal. [5]

(g) (i) Give an example of regular languages. [1]

(ii) If L and M are regular languages then prove that L - M is also regular languages. [4]

(h) Design a push down automata (PDA) to accept the set of all strings of 0's and 1's such that no prefix has more 1's than 0's. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) In a Boolean algebra *B*, prove that for *a, b, c* in *B*, [3]

$(a+b+c).(a'+b+c).(a+b'+c).(a+b+c') = (b+c).(c+a).(a+b).$

(ii) Prove that there does not exist a Boolean algebra containing only three elements. [4]

(iii) Show that the number of elements in a finite Boolean algebra is of power of 2. [3]

(b) (i) Show that the union two recursive languages is recursive. [3]

(ii) Show that the union of two recursive enumerable languages is also recursively enumerable. [3]

(iii) Show that any non trivial property of the recursively enumerable language is undecidable. [4]

(c) (i) Define turning machine and explain programming techniques for turning machine. [4]

(ii) Design a turning machine to accept a palindrome. [3]

(iii) Write a note on variants of a turning machine. [3]

(d) (i) State and proof Pumping lemma. [4]

(ii) Define context free language. [2]

(iii) Using Pumping lemma, show that language $L = \{a^n b^n c^n : n \geq 1\}$ is not a context free language. [4]

(e) (i) Prove that in a Boolean algebra, $a + a = a$ and $a.a = a$. [3]

(ii) When is a Boolean algebra said to atomic. Give an example of it. [3]

(iii) Show that a lattice is distributive iff following identity holds [4]

$(x \cap y) \cup (y \cap z) \cup (z \cap x) = (x \cup y) \cap (y \cup z) \cap (z \cup x).$