B.A/B.Sc 3rd Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH3CC07 (Numerical Methods)

Time: 2 Hours — Full Marks: 40

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

Answer any eight questions: $8\times5 = 40$

1. (i) Define relative error. [1]

   (ii) Obtain the relative error for computation of $u = x_1^{m_1} x_2^{m_2} x_3^{m_3} \ldots x_n^{m_n}$ in terms of the relative errors of $x_1, x_2, x_3, \ldots, x_n$ [4]

2. Prove that the sum of Lagrange's coefficients is unity. [5]

3. (i) Establish Newton-Raphson's iterative method geometrically. [3]

   (ii) Obtain an iterative formula to find the p-th root of a. [2]

4. Find the inverse of the matrix $\begin{pmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 1 \end{pmatrix}$ by Gauss-Jordan method. [5]

5. Using the LU-decomposition method, solve the following system of equations, $x + 2y + 3z = 14$, $2x + 5y + 2z = 18$, $3x + y + 5z = 20$. [5]

6. (i) If $f(x) = x^2$, then show that $\Delta^r f(x) = 0$ for $r \geq 3$ where Δ is the forward difference operator. [2]

   (ii) Prove that $\nabla^n y_k = \Delta^n y_{k-n}$ where Δ is the forward difference operator and ∇ is the backward difference operator. [3]

7. If $f(x) = a + bx + cx^2$, prove that $\int_1^2 f(x)dx = \frac{1}{12}[f(0) + 22f(2) + f(4)]$. [5]

8. Define degree of precision of a quadrature formula. Prove that Simpson's one-third rule is exact for all polynomials of degree not exceeding 3. [2+3]

9. (i) Solve the Initial Value Problem: $\frac{dy}{dx} = \frac{x^2}{1+y^2}$, $y(0) = 0$ by successive approximation method to obtain $y(0.25)$ correct upto three decimal places. [3]

   (ii) Give the geometrical interpretation of Euler's method for solving the Initial Value Problem: $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$. [2]

10. Determine the largest eigen value and the corresponding eigen vector of the mairix $A = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 1 & 4 \end{pmatrix}$ by Power method. [5]