B.A/B.Sc 1st Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH1CC01 (Calculus, Geometry and Differential Equation)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 5×6 = 30

(a) (i) Prove that $sinh^{-1}x = \log(x + \sqrt{x^2+1})$, for all $x \in \mathbb{R}$. [2+3]

(ii) Find the points of inflexion, if any, of the curve $y = e^{-x^2}$.

(b) (i) Trace the curve $x = y(y^2 - 1)$. [3+2]

(ii) Find the nature of concavity of the curves (i) $y = x^4$ and (ii) $y = e^x$.

(c) (i) Evaluate $\int_0^{\frac{\pi}{4}} tan^5 x dx$ by reduction formula. [2+3]

(ii) Derive a reduction formula for $\int x(logx)^n dx$ for $n \in \mathbb{Z}^+$.

(d) (i) Find the parametric equations of (i) $y = \cosh\left(\frac{x}{5}\right)$ (ii) $x^2 - y^2 = 4$. [2+3]

(ii) State Wallis's formula and hence evaluate $\int_0^{\frac{\pi}{2}} sin^7\theta cos^8\theta d\theta$.

(e) (i) Prove that the linear part of the equation [2+3]

$$4x^2 - 12xy + 9y^2 + 4x + 6y + 1 = 0$$

cannot be made to disappear by only change of parallel axes.

(ii) Prove that the length of the focal chord of the conic $\frac{l}{r} = 1 - ecos\theta$, which is inclined to the initial line at an angle α, is $\frac{2l}{1-e^2cos^2\alpha}$.

(f) (i) Find the equations of the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point $(1,1,1)$. [3+2]

(ii) Find the equation of the right circular cylinder whose axis is $x = y = z$ and radius is 5 units.

(g) (i) Solve: $2ydx - xdy = xy^3dy$. [2+3]

(ii) Solve $p = \sin(y - px)$, $p \equiv \frac{dy}{dx}$ for general and singular solutions.

(h) (i) Solve: $sinx\frac{dy}{dx} + y^2 = ycosx$. [3+2]

(ii) Solve: $(y^2e^{xy^2} + 4x^3)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$.

2. Answer any three questions: 10×3 = 30

(a) (i) If $x = \tan(logy)$ then prove that [3+3+4]

$(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0.$

(ii) Evaluate $\lim_{x\to 0}\left(\frac{tanx}{x}\right)^{\frac{1}{x^2}}$.

(iii) Find the asymptotes to the curve,

$x^4 - 5x^2y^2 + 4y^4 + x^2 - 2y^2 + 2x + y + 7 = 0.$

(b) (i) Find the entire surface area of the solid formed by the revolution of the cardioide $r = a(1+cos\theta)$ about the initial line. [4+3+3]

(ii) If $I_n = \int_0^{\frac{\pi}{2}} x^n sinxdx$ and $n > 1$ then show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

(iii) Find the length of the curve $x = e^{\theta}\sin\theta,\ \ y = e^{\theta}cos\theta$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$.

(c) (i) Reduce the following to the canonical form [4+4+2]

$8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0.$

(ii) A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0, z = 0$ and passes through a fixed point $(0,0,c)$. If the section of the cone by the plane $x = 0$ is a rectangular hyperbola then prove that the vertex lies on the fixed circle

$x^2 + y^2 + z^2 + 2ax + 2by = 0, 2ax + 2by + cz = 0.$

(iii) Find the equations to the generating lines of the hyperboloid

$\frac{1}{4}x^2 + \frac{1}{9}y^2 - \frac{1}{16}z^2 = 1$ which pass through the point $\left(2,-1,\frac{4}{3}\right)$.

(d) (i) Solve: $(x^2y^3 + 2xy)dy = dx$, given that when $x = 1, y = 1$. [3+3+4]

(ii) Using the transformation $x^2\sqrt{y} = v$, solve

$\left(2 + 2x^2\sqrt{y}\right)ydx + \left(x^2\sqrt{y} + 2\right)xdy = 0.$

(iii) By the substitution $x^2 = u, y^2 = v$ (or, otherwise) reduce the equation $x^2 + y^2 - (p + p^{-1})xy = c^2$ to Clairaut's form and find the general integral and singular solution.

(e) (i) If the astroid $x^{2/3} + y^{2/3} = c^{2/3}$ is the envelope of the family of ellipses $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ then prove that $a + b = c$. [4+3+3]

(ii) Find the length of the parabola $x^2 = 20y$ measured from the vertex to an extremity of its latus rectum.

(iii) Find the equation of a circle passing through the points $(2,-1,-3)$, $(1,1,-3)$, $(-1,5,0)$.