B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5DSE11 (Linear Programming)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.
Candidates are required to write their answers in their own words as far as practicable.
[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Solve the following LPP: [5]
Maximize $Z = 3x_1 + 5x_2$
subject to the constraints

$$x_1 - 2x_2 \leq 6,$$
$$x_1 \leq 10,$$
$$x_2 \geq 1$$

and $x_1, x_2 \geq 0$.

(b) Prove that the objective function of a linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solution. [5]

(c) Let $x_1 = 2, x_2 = 3, x_3 = 1$be a feasible solution of the LPP [5]

Maximize $Z = x_1 + 2x_2 + 4x_3$
subject to $2x_1 + x_2 + 4x_3 = 11,$
$3x_1 + x_2 + 5x_3 = 14$

and$x_1, x_2, x_3 \geq 0$.
Find a basic feasible solution.

(d) Use the dual simplex method to solve the LPP given below: [5]
Maximize $Z = -3x_1 - 2x_2$
subject to $x_1 + x_2 \leq 7,$
$$x_1 + 2x_2 \geq 10$$
and $x_1, x_2 \geq 0$.

(e) In an assignment problem, if a constant is added or subtracted to every element of any row (or column) of the cost matrix $\left[c_{ij}\right]$, then prove that an assignment that minimizes the total cost on one matrix also minimizes the total cost on the other matrix. [5]

(f) Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method. [5]

	D_1	D_2	D_3	a_i
O_1	4	8	8	66
O_2	16	24	16	72
O_3	8	16	24	77
b_j	72	102	41	

(g) Solve the following travelling salesman problem: [5]

		To			
		A	*B*	*C*	*D*
From	*A*	∞	7	6	8
	B	7	∞	8	5
	C	6	8	∞	9
	D	8	5	9	∞

(h) Using dominance property reduce the following payoff matrix to 2×2 matrix and hence solve the problem: [5]

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	1	2	-2	2
	A_2	3	1	2	3
	A_3	-1	3	2	1
	A_4	-2	2	0	-3

2. Answer any three questions: 3×10 = 30

(a) (i) If for a basic feasible solution X_B of a linear programming problem: [6]

Maximize $Z = CX$

subject to $AX = b, X \geq 0$,

$Z_j - C_j \geq 0$ for every column a_j of A, then prove that X_B is an optimal solution.

(ii) Find the basic solutions of the system of equations, $2x_1 + x_2 + 4x_3 = 11, 3x_1 + x_2 + 5x_3 = 14$. [4]

(b) Solve the following LPP by two phase method: [10]

Maximize Z = $2x_1 - 3x_2$

subject to

$$-x_1 + x_2 \geq -2,$$
$$5x_1 + 4x_2 \leq 46$$
$$7x_1 + 2x_2 \geq 32$$
$$\text{and } x_1, x_2 \geq 0.$$

(c) (i) If the *i*-th constraint of the primal problem is an equation then show that the*i*-th variable of the corresponding dual problem is unrestricted in sign. [4]

(ii) Using duality, solve the following LPP [6]

Minimize Z = $10x_1 + 6x_2 + 2x_3$

subject to

$$-x_1 + x_2 + x_3 \geq 1,$$
$$3x_1 + x_2 - x_3 \geq 2$$
$$\text{and} x_1, x_2, x_3 \geq 0.$$

(d) (i) Prove that the number of basic variables in a transportation problem with *m* origins and *n* destinations is at most *m*+*n*-1. [5]

(ii) Solve the following assignment problem: [5]

	M_1	M_2	M_3	M_4
J_1	10	24	30	15
J_2	16	22	28	12
J_3	12	20	32	10
J_4	9	26	34	16

(e) (i) Solve the following game: [5]

Player B

Player A	B_1	B_2	B_3	B_4
A_1	19	6	7	5
A_2	7	3	14	6
A_3	12	8	18	4
A_4	8	7	13	-1

(ii) State and prove fundamental theorem of rectangular games. [5]

B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)
Subject: Mathematics
Course: BMH5DSE12 (Number Theory)

Time:3 Hours Full Marks: 60

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Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Show that $4(29)! + 5$ is divisible by 31. [5]

(b) Prove that no prime factor of $n^2 + 1$ can be of the form $4m - 1$ where m is an integer. [5]

(c) Prove that $\varphi(3n) = 3\varphi(n)$ if and only if 3 is a divisor of n, where φ is the Euler's phi function. [5]

(d) If p is a prime number then show that $(p-1)! \equiv p - 1(mod(1 + 2 + 3 + \cdots \ldots\ldots + (p-1)))$. [5]

(e) If $gcd(a,n) = gcd(b,n) = gcd\left(ord_n^a, ord_n^b\right) = 1$ then show that $ord_n^{ab} = ord_n^a . ord_n^b$. [5]

(f) Solve $25x \equiv 15(mod\ 29)$. [5]

(g) Find the least positive residue in 2^{41}modulo 23. [5]

(h) Find the general solution in integer of the equation $7x + 11y = 1$. [5]

2. Answer any three questions: 10×3 = 30

(a) (i) How many primitive roots are there in modulo12^{100}? [5]

(ii) Find the order of 12 modulo 25. [5]

(b)	(i)	If p and p^2+8 are both primes, prove that $p=3$.	[5]
	(ii)	Prove that the total number of positive divisors of a positive integer n is odd if and only if n is a perfect square.	[5]
(c)	(i)	Let the integer a have order k modulo n. Show that $a^h \equiv 1 (mod n)$ if and only if k/h.	[3]
	(ii)	Prove that the functions τ and σ are both multiplicative.	[7]
(d)	(i)	Find four consecutive integers divisible by 3,4,5,7 respectively.	[7]
	(ii)	If $gcd(a,m)=1$, then prove that the linear congruence $ax \equiv b (mod m)$ has a unique solution.	[3]
(e)	(i)	Prove that n is divisible by 19 if $a+2b$ is divisible by 19 where $n=10a+b$.	[5]
	(ii)	Find the reminder when $1^5+2^5+\cdots+100^5$ is divided by 5.	[5]

B.A/B.Sc 5th Semester (Honours) Examination, 2020 (CBCS)

Subject: Mathematics

Course: BMH5DSE13 (Point Set Topology)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a)	If u, v, w are cardinal numbers, prove that $\left(u^v\right)^w = u^{vw}$.	[5]
(b)	Define an ordinal number. State and prove the principle of transfinite induction.	[1+4]
(c)	Define Kuratowski closure operator and explain the topology derived from it.	[1+4]
(d)	Let (X,τ) be a topological space. Prove that X is connected if and only if there is no continuous surjective map $f: X \to \{0,1\}$, where $\{0,1\}$ is the two point discrete space.	[5]
(e)	Define a locally compact space and prove that every closed subspace of a locally compact space is locally compact.	[5]
(f)	Prove that every totally bounded metric space is bounded. Is the converse true? Justify your answer.	[2+3]
(g)	Prove that a topological space (X,τ) is locally connected if and only if each component of an open subspace is open in (X,τ).	[5]
(h)	Prove that the union of an arbitrary family of connected sets, no two of which are separated, is a connected set.	[5]

2. Answer any three questions: 3×10 = 30

(a) (i) Let u be the cardinal number of the set U. Prove that the cardinal number of the power set $P(U)$ of U is 2^u. [4]

(ii) Prove that every non-degenerate interval open, closed or semi open and semi closed has the same cardinality as that of $\mathbb{R}$. [3]

(iii) Let $(A,\le)$ be a totally ordered set. Define an initial segment A_x of A. If $x \le y$ in A, prove that $A_x \subset A_y$. [3]

(b) (i) Give an example to show that there exists continuous mapping of a topological space into a topological space which is neither open nor closed. [5]

(ii) If $A \subset X, X$ a topological space, show that $\left(\overline{X-A}\right) = X - A^\circ$ and $(X-A)^\circ = X - \overline{A}$, where the symbols have their usual meanings. [3+2]

(c) (i) Let X be a topological space such that all real valued continuous function on X satisfy intermediate value property. Prove that X is connected. [3]

(ii) Construct a real valued function on a connected space which satisfies intermediate value property but not continuous. [3]

(iii) Give an example of a connected space which is not locally connected. [4]

(d) (i) Prove that the image of a locally connected space under a mapping f which is both open and continuous is locally connected. [5]

(ii) Show that every closed bounded interval in the real number space $(\mathbb{R}, \tau)$ with usual topology is compact. [5]

(e) (i) Prove that a real valued continuous function defined on a compact topological space is bounded and attains its least and greatest values. [5]

(ii) Prove that a metric space (X,d) is compact if and only if every family of closed sets with the finite intersection property has nonempty intersection. [5]